Technical Comments

Comments on "Effects of Rotary Inertia on the Supersonic Flutter of Sandwich Panels"

LARRY L. ERICKSON*
NASA Ames Research Center, Moffett Field, Calif.

MARAFIOTI and Johnston[1] recently extended the sandwich panel flutter analysis of Refs. 2 and 3 to include the effects of rotary inertia. Unfortunately, they represented the rotary inertia moments in a manner that is not consistent with the assumed panel deformations. The sandwich panel theory employed in Ref. 1 is formulated in terms of the transverse displacement (w) and two transverse shear angles ($\gamma_x = Q_x/D_Q$, $\gamma_y = Q_y/D_Q$). The shear angles and the rotations of the panel cross sections both contribute to the panel slope, (e.g., $w_{,x} = \gamma_x + \alpha_x$). The quantity $\alpha_x = w_{,x} - \gamma_x$ is the angle of rotation in the xz-plane, of a line element originally perpendicular to the panel's undeformed middle surface (see Appendix C of Ref. 4 or Fig. 18 of Ref. 5). As implied by Eq. (C3) of Ref. 4, the virtual work of the moments (both elastic and inertia) is associated with the angle of rotation rather than with the angle defined by the panel slope.

Inclusion of the inertia moment due to mass being accelerated through the angle α_x, in the corresponding moment equilibrium equation of Ref. 4 yields

$$\frac{Q_x}{D} = -w_{,xyy} + \frac{1-u}{2}\frac{Q_{x,yy}}{D_Q} + \frac{1+u}{2}\frac{Q_{y,xy}}{D_Q} - \left(w_{,x} - \frac{Q_x}{D_Q}\right)_{,xx} + \frac{J}{D}\left(w_{,x} - \frac{Q_x}{D_Q}\right)_{,tt}$$

This equation differs from Eq. (2) of Ref. 1 only in the rotary inertia term, which Marafioti and Johnston write as $(J/D) \times (w_{,x})_{,tt}$. Use of the panel rotation, rather than the panel slope, in the rotary inertia term leads to results which are fundamentally different from those presented in Ref. 1. For example, the characteristic equation of Ref. 1 is changed to

$$(\bar{m}^4 - 4\gamma\bar{m}^3 + \pi^2\bar{A}\bar{m}^2 + \bar{\lambda}\bar{m} - \pi^4\bar{B}) \times \left\{1 - \frac{r}{\theta^2}\left[\left(\frac{\bar{m}^2}{\pi^2} - n^2\theta^2\right)\left(\frac{1-u}{2}\right) + \langle\theta^2\bar{J}\Omega^2\rangle\right]\right\} = 0 \quad (1)$$

where

$$\gamma = \lambda\theta r/[4\pi^2(1 - rk_x)] \quad (2a)$$

$$\bar{A} = [\theta^2/(1 - rk_x)]\,[k_x - 2n^2 + r(\Omega^2 + n^2\bar{N}_y + n^2k_x) + \bar{J}\Omega^2 - \langle r\bar{J}\Omega^2k_x\rangle] \quad (2b)$$

$$\bar{\lambda} = [\lambda\theta^3/(1 - rk_x)](1 + n^2r - \langle r\bar{J}\Omega^2\rangle) \quad (2c)$$

$$\bar{B} = [\theta^4/(1 - rk_x)][(\Omega^2 + n^2\bar{N}_y)(1 + n^2r - \langle r\bar{J}\Omega^2\rangle) - n^2(n^2 - \bar{J}\Omega^2)] \quad (2d)$$

The notation used above is the same as used by Marafioti and Johnston except that their m has been written here as $\bar{m}$ to distinguish the characteristic roots from the m (number of half-waves) appearing in their in-vacuo frequency equation. The terms in Eqs. (1) and (2) not contained in their results have been enclosed in angular brackets $\langle\ \rangle$.

Application of the simple support boundary conditions used in Ref. 1 (at the panel's leading and trailing edges), yields

$$(1 - r\bar{J}\Omega^2)^2(e^{\bar{m}_6} - e^{\bar{m}_5})F(\alpha,\delta,\epsilon,\gamma) = 0 \quad (3)$$

where $\bar{m}_5$ and $\bar{m}_6$ are the two roots of the quadratic part of Eq. (1) and

$$F(\alpha,\delta,\epsilon,\gamma) = [(\delta^2 + \epsilon^2)^2 + 4\alpha^2(\delta^2 - \epsilon^2) + 4\gamma^2(4\alpha^2 + \delta^2 - \epsilon^2)]\sin\delta\,\sinh\epsilon - 8\epsilon\delta(\alpha^2 - \gamma^2)(\cosh\epsilon\cos\delta - \cosh 2\alpha) \quad (4)$$

The quantities α, δ, ϵ, and γ are the same as in Refs. 1 and 3 and determine the four roots of the quartic part of Eq. (1). Equation (4) corresponds to Eq. (23) of Ref. 1 which contains three typographical errors.

Equation (3) is not valid for $1 - r\bar{J}\Omega^2 = 0$ since this condition corresponds to repeated characteristic roots [given by $(\bar{m}^2 - n^2\theta^2\pi^2)^2 = 0$]. For $\Omega^2 \gtrless 1/(r\bar{J})$, Eq. (3) is valid and becomes

$$F(\alpha,\delta,\epsilon,\gamma) = 0 \quad (5a)$$

$$e^{\bar{m}_6} - e^{\bar{m}_5} = 0 \quad (5b)$$

Equation (5b) and the quadratic part of Eq. (1) are satisfied for frequencies given by

$$\Omega^2 = 1/\bar{J}\{1/r + [(1 - \mu)/2][(m^2/\theta^2) + n^2]\} \quad (6)$$

where m and n are the number of half-wavelengths that form in the x- and y-directions, respectively. Eq. (6) gives the frequencies of the thickness-twist modes which, for a homogeneous panel, are described in Ref. 6. These frequencies are independent of λ (dynamic pressure); this is because the thickness-twist modes, for a simply supported panel with an isotropic core, occur without any transverse deflection (i.e., $w(x,y,t) = 0$). Hence, on the basis of the inviscid aerodynamic theory employed, no aerodynamic forces are produced. For other boundary conditions, or for panels having orthotropic cores such as honeycomb, the thickness-twist frequencies are not independent of dynamic pressure.[5]

The simultaneous satisfaction of Eq. (5a) and the quartic part of Eq. (1), for various values of λ and Ω^2, determine frequency loops formed by the bending and thickness-shear fre-

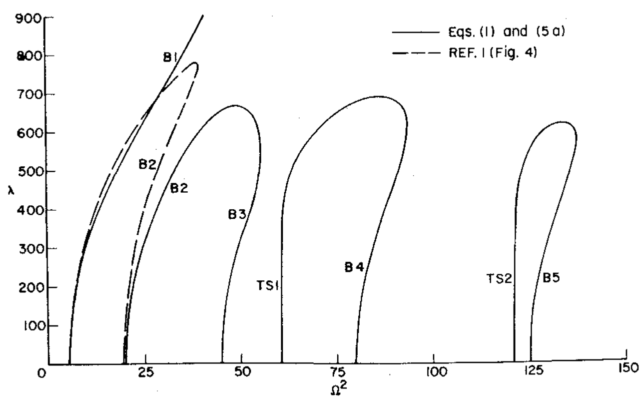

Fig. 1 Frequency loops for $\bar{J} = 0.05$; $r = 1$, $k_x = -4$, $\bar{N}_y = 0$, $n = \theta = 1$.

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* Aerospace Scientist, Structural Dynamics Branch.

quencies. For no airflow, these frequencies are given by

$$\Omega^2 = 2[\bar{\beta}^2 - \bar{p}(1 + r\bar{\beta})]/[1 + (r + \bar{J})\bar{\beta} - r\bar{J}\bar{p} \pm ([1 + (r - \bar{J})\bar{\beta} + r\bar{J}\bar{p}]^2 + 4\bar{J}\bar{\beta})^{1/2}] \quad (7)$$

where

$$\bar{\beta} = (m/\theta)^2 + n^2 \qquad \bar{p} = (m/\theta)^2 k_x + n^2 \bar{N}_y$$

The frequencies given by the smaller solution for Ω^2 are referred to as the bending set of frequencies because for $r = \bar{J} = 0$ (shear flexibility and rotary inertia neglected) they reduce to the frequencies of the pure bending motion given by classical plate theory; the large solution for Ω^2 gives the frequencies of the thickness-shear set of modes. (Both sets are described in Ref. 6.) In the case of a beam, the bending and thickness-shear sets of frequencies are both predicted by Timoshenko's beam theory[7] and have been discussed in Ref. 8.

Some numerical results obtained from the solutions given by Eq. (5a) and the quartic part of Eq. (1) are shown by the solid curves in Figs. 1 and 2 for $\bar{J}$ equal to 0.05 and 0.50, respectively, and with $r = 1$, $k_x = -4$, $\bar{N}_y = 0$, $n = \theta = 1$ in both cases. (Bm and TSm denote the mth bending and thickness-shear frequencies, respectively.) For comparison, results presented in Fig. 4 of Ref. 1 are shown by the dashed curves. The analysis of Ref. 1 predicts that the first two bending frequencies coalesce whereas the present analysis shows that $B1$ and $B2$ are uncoupled for the parameter values selected. This change in frequency coalescence behavior is due to the presence of the TS frequencies and is discussed in greater detail in Ref. 5. (The thickness-shear and the thickness-twist frequencies are not predicted by the analysis of Ref. 1.)

An additional effect of the rotary inertia is that it makes the solution for λ_{cr} dependent on $\bar{N}_y$. This differs from the solution of Ref. 3 ($\bar{J} = 0$) where the frequency and the crossflow loading term always grouped in the characteristic equation as $\Omega^2 + n^2\bar{N}_y$, a change in $\bar{N}_y$ merely shifting the frequency loops along the Ω^2 axis. This unique grouping does not occur for $\bar{J} > 0$ [see Eq. (1)]. Numerical results in Ref. 5 indicate, however, that $\bar{N}_y$ has a relatively small effect on frequency coalescence.

It should also be noted that values for $\bar{J} > 0.01$ are probably not representative of typical sandwich construction.† For panels having face sheets that are thin compared to the core thickness (an implicit assumption in the panel theory of Ref. 4 since the face sheet bending stiffness is neglected), $\bar{J}$ is given, approximately, by

$$\bar{J} \approx (\pi/2)^2(h/b)^2[1 + \tfrac{1}{2}(\rho_c h/\rho_s t_s)]/[1 + (\rho_c h/\rho_s t_s)]$$

where h is the core thickness and $(\rho_c h)/(\rho_s t_s)$ is the ratio of the core weight to the weight of the two face sheets (per unit area). Thus, $\bar{J} = 0.5$ (Fig. 2) requires the core thickness to be of the same order of magnitude as the panel width b.

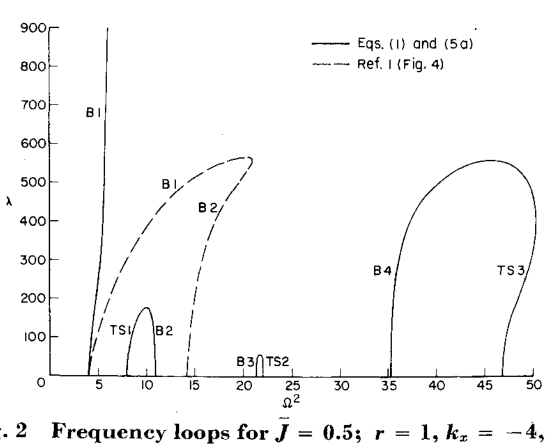

Fig. 2 Frequency loops for $\bar{J} = 0.5$; $r = 1$, $k_x = -4$, $\bar{N}_y = 0$, $n = \theta = 1$.

† Results are presented in Ref. 5 for the range $0 \leq \bar{J} \leq 0.01$. (In Ref. 5, $\bar{J}$ is called χ.)

References

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Reply by Authors to L. L. Erickson

Frank A. Marafioti* and E. Russell Johnston Jr.†
University of Connecticut, Storrs, Conn.

ERICKSON claims that an inconsistency exists in Ref. 1 because the effect of shear on rotary inertia was omitted. By including the terms $[J/D(Q_x/D_Q)]_{,tt}$ and $[J/D(Q_y/D_Q)]_{,tt}$, Erickson has extended the work of Ref. 1 to include the effect of shear on rotary inertia. It was pointed out in Ref. 1 that the effect of transverse shear on the rotary inertia term was neglected (p. 246, second column, Ref. 1). The authors feel that the rotary inertia moments are presented in a consistent manner. Equation (C3) of Ref. 2 discusses the boundary conditions that are to be applied to a typical sandwich panel and does not imply that transverse shear must be considered for rotary inertia effects. The stress resultants and couples used in Ref. 1 consider the effect of transverse shear; therefore, the correct panel deformation patterns are obtained and shear behavior is considered.

In Ref. 3, Erickson and Anderson did not include the effects of rotary inertia in solving the flutter problem. In Ref. 1, Marafioti and Johnston include the effects of the rotary inertia terms $(J/D)w_{,xtt}$ and $(J/D)w_{,ytt}$. The results obtained in Ref. 1 show that in all cases the effect of these additional terms is to lower the critical dynamic pressure by 10 to 20% (Figs. 5 and 6, p 249, Ref. 1). Erickson now includes an addition term in the rotary inertia expressions, which become

$$J/D[w_{,x} - (Q_x/D_Q)]_{,tt} \text{ and } J/D[w_{,y} - (Q_y/D_Q)]_{,tt}$$

His results show differences of as much as 70% when compared to Ref. 1, and even larger differences from the results presented in Ref. 3. In Ref. 3, the dynamic pressure was calculated to be approximately 1050 (Fig. 5, Ref. 1); whereas,

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* NASA fellow, Civil Engineering Department.
† Professor of Civil Engineering.